VOLUME-10 Part B and C

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IV. Quantum Mechanics

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IV.1.Wave function and Operators

1.1. Introduction

According to Rutherford model of an atom, a tiny, massive, positively charged nucleus surrounded at a relatively great distance by enough electrons in a circular orbit.

Therefore, the centripetal force is,

$$F_c = \frac{mv^2}{r} \quad \text{.......(where } v = \text{velocity of electron)}$$

Centripetal force helds the electrons in an orbit of radius r from the nucleus, is provided by electrostatic force.

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

The condition for stability is,

$$F_c = F_e$$

$$\therefore \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\therefore \quad mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

$$\therefore \quad v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mr}$$

$$\therefore \quad v = \frac{e}{\sqrt{4\pi\varepsilon_0 rm}}$$

The total energy E of an e^- in hydrogen is sum of $k.\varepsilon$ and $P.\varepsilon$.

$$\therefore \quad k.\varepsilon = \frac{1}{2}mv^2$$

$$\text{and} \quad P.\varepsilon = \frac{-e^2}{4\pi\varepsilon_0 r} \quad \text{(negative sign indicates electron is bound to nucleus)}$$

$$\therefore \quad E = k.\varepsilon + P.\varepsilon$$

$$\therefore \quad E = \frac{1}{2}mv^2 + \left(-\frac{e^2}{4\pi\varepsilon_0 r}\right)$$

$$\therefore \qquad E = \frac{1}{2} m \frac{e^2}{4\pi\varepsilon_0 mr} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\therefore \qquad E = \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right) - \left(\frac{e^2}{4\pi\varepsilon_0 r}\right)$$

$$\therefore \qquad E = -\frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_0 r}\right)$$

$$\therefore \qquad E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Therefore, the total energy of an atomic electron is negative, if it is bound to the nucleus. If E is greater than zero ($E > 0$), the electron would have too much energy to remain in a closed orbit about the nucleus.

From above equation

$$E = -\frac{e^2}{8\pi\varepsilon_0 r}$$

Here, $r = 5.3 \times 10^{-11} m$, $e = 1.6 \times 10^{-19} C, \pi = 3.14,$ $\varepsilon_0 = 8.85 \times 10^{-12} \frac{c^2}{vm^2}$

In above equation $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$

$$\therefore \qquad E = -13.6 \times 1.6 \times 10^{-19} J$$

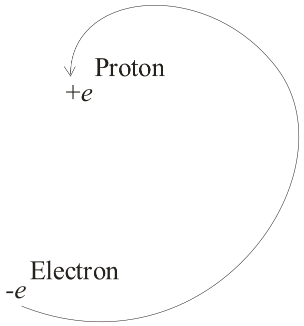
$$\therefore \qquad E = \frac{-13.6 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-19}} eV \; [\because eV = 1.6 \times 10^{-19} J\,]$$

$$\therefore \qquad E = -13.6\, eV$$

According to electromagnetic theory, accelerated electric charge radiates energy in the form of electromagnetic waves.

As electron loses energy, E becomes more negative. As r decreases, the electron radiates energy faster and it spirals in toward the nucleus. Only 10^{-16} sec. would be required for stable hydrogen atom to collapse. But according to electromagnetic theory, yet atoms do not collapse.

This contradiction says, law of physics is valid in macroscopic world but not in microscopic world.



To verify the classical calculation to be approximately correct, we note that the de-Broglie wavelength of an α-particle where speed is $2\times10^{7}\,m/s$ is,

$$\lambda=\frac{h}{mv}$$

$$\lambda=\frac{6.63\times10^{-34}}{6.6\times10^{-27}\times2\times10^{-7}}$$

$$\therefore\qquad \lambda=5\times10^{-15}\,m$$

The closet α-particle with this wavelength ever gets to a gold nucleus is $3\times10^{-14}\,m,$ which is 6 de-Broglie wavelength.

Rutherford's classical formula to be at least similar to a quantum-mechanical formula for the some process.

As a general rule, if the force between two particles varies as $r^{n},$ the cross-section for the scattering of one by the other varies as h^{4+2n}. But $F\alpha\frac{1}{r^{2}}$, i.e. $F\alpha\, r^{-2}$

$$\therefore\qquad h^{4+2n}=h^{4-4}=h^{0}=1$$

Therefore, cross-section is independent of planck's constant *h*.

1.2. Atomic Spectra

The heated solids emits radiations in which all wavelengths are present, with different intensities. When solid is heated to incandescence, we are witnessing the collective behaviour of a great many interacting atoms rather than the characteristic behaviour of the individual atoms of a particular elements.

The spectrum of an excited molecular gas or vapour contains 'bands' which consist of many separate lines very close together. Bands are their origin to rotations and vibrations of the atoms in an electronically excited molecule. When white light is passed through a gas, it is found to absorb light of certain of the wavelengths present in its emission spectrum.

The resulting absorption line spectrum consists of a bright background crossed by dark lines corresponding to the missing wavelengths. The emission spectra consists of bright lines on a dark background. The dark Fraunhofer lines in the solar spectrum occur because the luminous part of the sun, which radiates almost exactly according to theoretical predictions for any object heated to 5800^0K, is surrounded by an envelope of cooler gas which absorbs light of certain wavelengths only. The wavelengths present in atomic spectra fall into definite sets called 'spectral series'. The wavelengths in each series can be specified by a simple empirical formula. The first such spectral series was found by J. J. Balmer in 1885 of a study of the visible part of the hydrogen spectrum.

Figure 1 shows the Balmer series, the line with longest wavelength, 6,563 A^o, is designated by H_α and wavelength 4,863 A^o by H_β and so on. As wavelength decreases, the lines are found closer together and weaker in intensity until the series limit at 3,646 A^o is reached beyond which there are no separate lines but only a faint continuous spectrum.

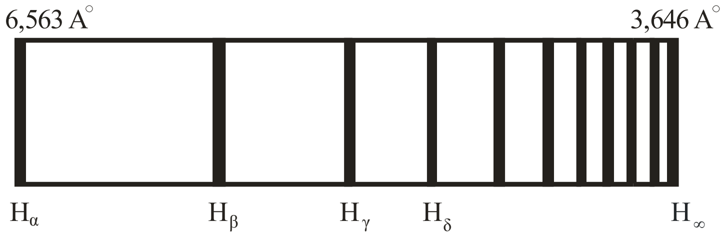


Figure 1

Bolmer's formula for the wavelength of this series is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), n = 3,4,5,.... \text{Balmer}$$

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