VOLUME-09 Part B and C

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III.6. Electromagnetic Waves in free space, dielectrics and conductors & Reflection and refraction, polarization

6.1. Electromagnetic Waves in Matter

1. Propagation in Linear Media:

Inside matter, but in regions where there is no free charge or free current, Maxwell's equations become

$$\left.\begin{aligned} &(i)\nabla\cdot\mathbf{D}=0, \qquad (iii)\nabla\times\mathbf{E}=\frac{\partial\mathbf{B}}{\partial t},\\ &(ii)\nabla\cdot\mathbf{B}=0, \qquad (iv)\nabla\times\mathbf{H}=\frac{\partial\mathbf{D}}{\partial t},\end{aligned}\right\} \qquad \ldots 1$$

If the medium is linear,

$$\mathbf{D}=\mathbf{E}, \quad \mathbf{H}=\frac{1}{\mu}\mathbf{B} \qquad \ldots 2$$

And homogeneous (so $\in$ and μ do not vary from point to point), Maxwell's equations reduce to

$$\left.\begin{aligned} &(i)\nabla\cdot\mathbf{E}=0, \qquad (iii)\nabla\times\mathbf{E}=\frac{\partial\mathbf{B}}{\partial t},\\ &(ii)\nabla\cdot\mathbf{B}=0, \qquad (iv)\nabla\times\mathbf{B}=\mu\in\frac{\partial\mathbf{E}}{\partial t},\end{aligned}\right\} \qquad \ldots 3$$

Evidently electromagnetic waves propagate through a linear homogeneous medium at a speed

$$\upsilon=\frac{1}{\sqrt{\in\mu}}=\frac{c}{n}, \qquad \ldots 4$$

Where, $n\equiv\sqrt{\dfrac{\in\mu}{\in_0\mu_0}}$ = is the **index of refraction** of the material. …5

For most materials, μ is very close to μ_0, so

$$n\cong\sqrt{\in_r}, \qquad \ldots 6$$

where, $\in_r$ is the dielectric constant.

Since $\in_r$ is almost always greater than 1, light travels more slowly through matter — a fact that is well known from options.

All of our previous results carry over, with the simple transaction,

$\in_0\to\in$, $\mu_0\to\mu$ and hence $c\to\upsilon$.

The energy density is,

$$u = \frac{1}{2}\left(\in E^2 + \frac{1}{\mu} B^2 \right), \qquad ...7$$

And the Polynting vector is

$$\mathbf{S} = \frac{1}{\mu}(\mathbf{E} \times \mathbf{B}) \qquad ...8$$

For monochromatic plane waves the frequency and wave number are related by,

$$\omega = k\upsilon ,$$

the amplitude of **B** is,

$$\mathbf{B} = 1/\upsilon\, \mathbf{E}$$

and the intensity is,

$$I = \frac{1}{2} \in \upsilon E_0^2 . \qquad \ldots 9$$

As in the case of waves on a string, we expect to get a reflected wave and a transmitted wave.

$$\left.\begin{array}{ll} (i)\ \in_1 E_1^{\perp} = \in_2 E_2^{\perp}, & (iii)\ \mathbf{E}_1^{\parallel} = \mathbf{E}_2^{\parallel} \\ (ii)\quad B_1^{\perp} = B_2^{\perp}, & (iv)\ \frac{1}{\mu_1}\mathbf{B}_1^{\parallel} = \frac{1}{\mu_2}\mathbf{B}_2^{\parallel} \end{array}\right\} \qquad \ldots 10$$

These equations relate the electric and magnetic fields just to the left and just to the right of the interface between two linear media.

2. Reflection and Transmission at Normal Incidence:

Suppose the *xy* plane forms the boundary between two linear media. A plane wave of frequency ω, travelling in the *z* direction and polarized in the *x* direction, approaches the interface from the left.

$$\left.\begin{array}{l} \tilde{\mathbf{E}}_I(z,t) = \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \tilde{\mathbf{B}}_I(z,t) = \frac{1}{\upsilon_1} \tilde{E}_{0I} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{array}\right\} \qquad \ldots 11$$

It gives rise to a reflected wave,

$$\left.\begin{array}{l} \tilde{\mathbf{E}}_R(z,t) = \tilde{E}_{0R} e^{i(k_1 z - \omega t)} \hat{\mathbf{x}} \\ \tilde{\mathbf{B}}_R(z,t) = \frac{1}{\upsilon_1} \tilde{E}_{0R} e^{i(k_1 z - \omega t)} \hat{\mathbf{y}} \end{array}\right\} \qquad \ldots 12$$

which travels back to the left in medium (1), and a transmitted wave

$$\left.\begin{aligned}\tilde{\mathbf{E}}_T(z,t) &= \tilde{E}_{0T} e^{i(k_2 z-\omega t)}\hat{\mathbf{x}} \\ \tilde{\mathbf{B}}_T(z,t) &= \frac{1}{\upsilon_2}\tilde{E}_{0T} e^{i(k_2 z-\omega t)}\hat{\mathbf{y}}\end{aligned}\right\} \quad \ldots 13$$

which continues on the right in medium (2).

At $z=0$, the combined fields on the left, $\tilde{\mathbf{E}}_I + \tilde{\mathbf{E}}_R$ and $\tilde{\mathbf{B}}_I + \tilde{\mathbf{B}}_R$, must join the fields on the right, $\mathbf{E}_T$ and $\mathbf{B}_T$, in accordance with the boundary conditions.

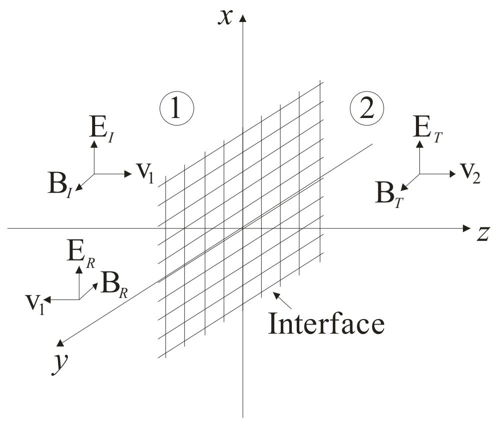

Figure 1

In case there are no components perpendicular to the surface, so (i) and (ii) are trivial. However, (iii) requires that

$$\tilde{E}_{0_I} + \tilde{E}_{0R} = \tilde{E}_{0T}, \quad \ldots 14$$

while (iv) says

$$\frac{1}{\mu_1}\left(\frac{1}{\upsilon_1}\tilde{E}_{0I} - \frac{1}{\upsilon_1}\tilde{E}_{0R}\right) = \frac{1}{\mu_2}\left(\frac{1}{\upsilon_2}\tilde{E}_{0T}\right), \quad \ldots 15$$

or

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \beta\tilde{E}_{0T}, \quad \ldots 16$$

where, $$\beta \equiv \frac{\mu_1\upsilon_1}{\mu_2\upsilon_2} = \frac{\mu_1 n_2}{\mu_2 n_1}. \quad \ldots 17$$

Equations (14) and (16) are easily solved for the outgoing amplitudes, in terms of the incident amplitude:

$$\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}, \qquad \tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}. \quad \ldots 18$$

These results are strikingly similar to the ones for waves on a string. Indeed, if the permittivities μ are close to their values in vacuum (as, remember, they are for most media),

Then

$$\beta = \upsilon_1 / \upsilon_2 ,$$

and we have

$$\tilde{E}_{0R} = \left(\frac{\upsilon_2 - \upsilon_1}{\upsilon_2 + \upsilon_1} \right) \tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2\upsilon_2}{\upsilon_2 + \upsilon_1} \right) \tilde{E}_{0I} \qquad ...19$$

In that case, as before, the wave is in phase (right side up) if $\upsilon_2 > \upsilon_1$ and out of phase (upside down) if $\upsilon_2 < \upsilon_1$; the real amplitudes are related by,

$$E_{0R} = \left| \frac{\upsilon_2 - \upsilon_1}{\upsilon_2 + \upsilon_1} \right| E_{0I}, \quad E_{0T} = \left(\frac{2\upsilon_2}{\upsilon_2 + \upsilon_1} \right) E_{0I} \qquad ...20$$

Or, in terms of the indices of refraction,

$$E_{0R} = \left| \frac{n_1 - n_2}{n_1 + n_2} \right| E_{0I}, \quad E_{0T} = \left(\frac{2n_2}{n_1 + n_2} \right) E_{0I} \qquad ...21$$

According to Equation (19), the intensity (average power per unit area) is,

$$I = \frac{1}{2} \in \upsilon\, E_0^2 .$$

If (again) $\mu_1 = \mu_2 = \mu_0$, then the ratio of the reflected intensity to the incident intensity is,

$$R \equiv \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}} \right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 , \qquad ...22$$

whereas the ratio of the transmitted intensity to the incident intensity is,

$$T \equiv \frac{I_T}{I_I} = \frac{\in_2 \upsilon_2}{\in_1 \upsilon_1} \left(\frac{E_{0T}}{E_{0I}} \right)^2 = \frac{4n_1 n_2}{(n_1 + n_2)^2} . \qquad ...23$$

R is called the **reflection coefficient** and *T* the **transmission coefficient;** they measure the fraction of the incident energy that is reflected and transmitted, respectively.

Notice that,

$$R + T = 1, \qquad ...24$$

as conversation of energy requires. For instance, when light passes from air $(n_1 = 1)$ into glass $(n_2 = 1.5)$. *R* = 0.04 and *T* = 0.96. Not surprisingly, most of the light is transmitted.

Continued with...Page 5 Onwards.... It's So Goooood!!!, Buy it now...!