VOLUME-08 Part B and C

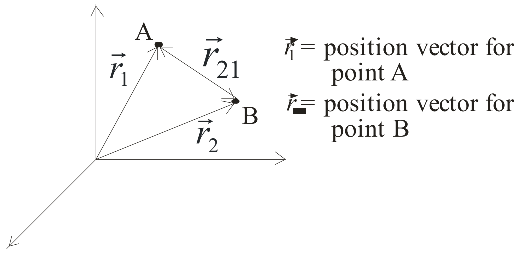
CONTENTS

III. Electromagnetic Theory

III.1. Electrostatics: Gauss's Law and its applications

1. Introduction

1.1 Gauss Law and Electrostatic Potential:



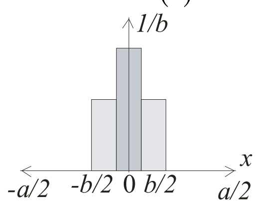
$\vec{r}_{21}$ = Vector from point A to B.

$\vec{r}_{12}$ = Vector from point B to A.

$\therefore \quad \vec{r}_{12} = \vec{r}_2 - \vec{r}_1 \qquad (\because \vec{r}_1 + \vec{r}_{12} = \vec{r}_2)$

i.e. the vector is in the direction that is in the reverse order of the subscripts.

1.2 Dirac Delta function (δ):



Area under rectangle is always unity. As width decreases, its height increases proportionally to maintain area 1.

The function δ is defined as the limiting case when width becomes zero and height becomes infinite, maintaining the area unity.

$$\delta(x) = 0 \quad \text{if} \quad x \neq 0$$
$$= \infty \quad \text{if} \quad x = 0$$

$$\therefore \quad \int_{-\infty}^{+\infty} \delta(x)\,.\,dx = 1$$

Similarly,

$$\delta(x-a) = 0 \quad \text{if} \quad x \neq a$$
$$= \infty \quad \text{if} \quad x = a$$

$$\therefore \quad \int_{-\infty}^{+\infty} \delta(x-a)\,.\,dx = 1$$

$\therefore$ From definition,

$$\int_{-\infty}^{+\infty} \delta(x-a)\, f(x)\, dx = f(a)$$

$$\therefore \quad \int_{-\infty}^{+\infty} \delta(x-a)\, f(x)\, dx = f(a) \quad if \quad -\infty \le a \le \infty$$
$$= 0 \qquad if \quad x = a$$

In 3-dimensions,

$$\delta^3(\vec{r}) = 0 \quad \text{if} \quad \vec{r} \neq \vec{0}$$

$$\int_{all\ space} \delta^3(\vec{r})\, dv' = 1$$

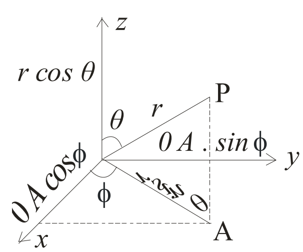
Similarly,

$$\int \delta^3(\vec{r} - \vec{r}\,') = 0 \quad \text{if} \quad \vec{r} \neq \vec{r}\,'$$
$$= \infty \quad \text{if} \quad \vec{r} = \vec{r}\,'$$

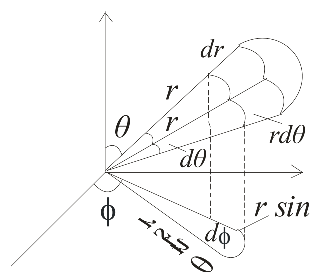
$\therefore$
$$\int_{all\ space} \delta^3(\vec{r} - \vec{r}\,')\, dv' = 1$$

$\therefore$
$$\int_{all\ space} \delta^3(\vec{r} - \vec{r}\,') f(\vec{r}\,')\, dv' = f(\vec{r}\,')$$

1.3 Spherical polar coordinates:



$z = r\cos\theta$
$y = r\sin\theta.\sin\phi$
$x = r\sin\theta.\cos\phi$



$\vec{dl_r} = dr\,.\hat{r}$
$\vec{dl_\theta} = r\,.\,d\theta.\hat{\theta}$
$\vec{dl_\phi} = r.\,\sin\theta.d\phi\,.\hat{\phi}$

$$\therefore d\,\vec{l} = d\,\vec{l}_r + d\,\vec{l}_\theta + d\,\vec{l}_\varphi$$
$$= dr.\hat{r} + r.d\theta.\hat{\theta} + r.\sin\theta.d\phi.\hat{\phi}$$
$$\therefore d\,v = d\,\vec{l}_r.d\,\vec{l}_\theta.d\,\vec{l}_\phi$$
$$= (\mathrm{dr}.\hat{\mathrm{r}})(\mathrm{r}.\mathrm{d}\theta.\hat{\theta})(\mathrm{r}\sin\theta.\mathrm{d}\phi.\,\hat{\phi})$$
$$= r^2\sin\theta.d\theta.d\phi$$

Area for constant

$$da = d\,l_\theta\,.\,d\,l_\phi$$
$$= (r.d\theta.\hat{\theta})(r\sin\theta.d\phi.\,\hat{\phi})$$
$$= r^2\sin\theta.d\theta.d\phi$$

$$\vec{\nabla}.v = \frac{\partial v}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial v}{\partial \theta}.\hat{\theta} + \frac{1}{r\ \sin\ \theta}.\frac{\partial v}{\partial \varphi}\hat{\varphi} \text{ ......(} v \text{ is a scalar quantity)}$$

1.4 Fundamental theorem of Gradient:

It states that, for scalar function T, the line integral of gradient of T between any two points a and b along any arbitrary curve c could be represented by the difference between T(b) and T(a).

$$\therefore \qquad \int_a^b \left(\vec{\nabla}\ T\right)\ d\vec{l} = T(b) - T(a)$$

1.5 Gauss divergence theorem:

It states that, the volume integral of divergence of vector A is equal to the surface integral of the normal component of vector A over a closed surface s that encloses the volume.

$$\int_v \left(\vec{\nabla}.\vec{A}\right) d\text{v} = \oint_s \vec{A}\ .\ \hat{n}\ ds$$

1.6 Stoke's theorem:

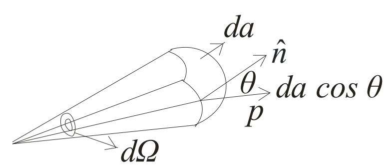
It states that, the surface integral of normal component of curl of vector A over an open surface S is equal to the line integral of vector A over the curve C bounding the surface.

$$\int_s \left(\vec{\nabla}\times\vec{A}\right)\hat{n}\ da = \oint_l \vec{A}\ .\ d\vec{l}$$

The value of $\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right)$:

$$\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\ \pi\pi^{\ 3}\left(\vec{r}\right)$$

1.7 Solid angle:



$$\therefore \qquad Solid\ angle = \frac{area\ substended\ by\ angle}{(radius)^2}$$

$$= \frac{da.\cos\theta}{r^2}$$

If point P is situated anywhere inside a closed curve subtended angle 2π with respect to the curve. If point P is situated anywhere inside a closed area subtends an angle 4π with respect to the surface.

1.8 Important Formulae

Cartesian :

$$dl = dx\ \hat{x}\ + dy\ \hat{y}\ +\ dz\ \hat{z};\ \ d\tau\ =\ dx\ dy\ dz$$

Gradient **:**

$$\vec{\nabla} t = \frac{\partial t}{\partial x}\hat{x} + \frac{\partial t}{\partial y}\hat{y} + \frac{\partial t}{\partial z}\hat{z}$$

Divergence **:**

$$\vec{\nabla} \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

*Curl***:**

$$\vec{\nabla} \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\hat{x} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\hat{y} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\hat{z}$$

Laplacian **:**

$$\nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

Spherical **:**

$$dl = dr\,\hat{r} + r d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi}; \qquad d\tau = r^2 \sin\theta\, dr\, d\theta\, d\phi$$

Gradient **:**

$$\vec{\nabla} t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \phi}\phi$$

Divergence **:**

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\, v_\theta\right) + \frac{1}{r\sin\theta}\frac{\partial v_\varphi}{\partial \varphi}$$

Curl:

$$\vec{\nabla} \times \vec{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}\left(\sin\theta\, v_\theta - \frac{\partial v_\varphi}{\partial \varphi}\right)\right]\hat{r}$$

$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial}{\partial r}\left(r v_\varphi\right)\right]\hat{\theta} + \left[\frac{\partial}{\partial r}\left(r v_\theta\right) - \frac{\partial v_r}{\partial \theta}\right]\hat{\varphi}$$

Laplacian :

$$\nabla^2 t = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial t}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 t}{\partial \phi^2}$$

Cylindrical**:**

$$dl = ds\,\hat{s} + s\, d\phi\,\hat{\phi} + dz\,\hat{z}; \qquad d\tau = s\, ds\, d\phi\, dz$$

Gradient :

$$\vec{\nabla} t = \frac{\partial t}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial t}{\partial \varphi}\hat{\phi} + \frac{\partial t}{\partial z}\hat{z}$$

Divergence:

$$\vec{\nabla} \cdot \vec{v} = \frac{1}{s}\frac{\partial}{\partial s}\left(s v_s\right) + \frac{1}{s}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z}$$

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