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II. Classical Mechanics

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II.1. Newton's Law of Motion:

According to Newton, any change in the motion of an object, described with respect to given reference, is the result of the mutual interaction between the object and its environment. The central problem of mechanics is to understand and quantify the connection between these interactions and the resulting motion. It is natural to expect that the interactions causing the motion can be quantified in terms of the measurable physical properties of the body and its environment, e.g. mass, electric charge, magnetic dipole moment, etc.

Newton gave a programme to attack this problem which comprises two steps:

1. A vector quantity called force (F) is regarded as the cause of change in the state of motions of a body or in other words, the vehicle of interaction between the moving object and its environment. The force acting on the body can cause acceleration (a) which is a vector quantity, like force.
2. The force acting on the body is calculated on the basis of the properties of the body and its environment, requiring determination of the appropriate force laws.

Newton's formulation of step (1) above forms the basis of his laws of motion. These laws of motion are valid in a class of reference frames called inertial frames. In fact, we can turn around and define the inertial frames to be those frames of reference in which Newton's laws of motion are valid.

Newton's laws can be stated as follows:

(i) Law of Inertia (1st Law):

In an inertial frame, every free particle has a constant velocity. The original version of this law written in Latin was slightly different. When translated into English, it reads: "every body free of impressed forces either preserves a state of rest or continues in uniform rectilinear motion *ad* infinitum."

In an inertial system a free particle undergoes equal displacement in equal intervals of time. This fact defines a time scale or a clock for inertial frames called inertial time scale.

Motion of free particle in inertial frames will be in straight lines. For, if this motion were on a curve with non-vanishing curvature, the velocity of this free particle, which is a vector tangent toe the path of the particle, would change with time, contradiction the first law. Thus, a path traced by a free particle in an inertial frame defines a straight line in that frame.

Since a free particle covers an equal measure of space in equal measure of time, *ad* infinitum, it implies that along the straight line of the path, space is uniform or homogenous, and so also time. Again, since the direction of the straight line path could have been any, it also implies that space is isotropic. So an inertial frame is also interpreted to stand for the homogeneity and isotropy of space, and homogeneity of time.

Since free particle travel in straight line of an ideal inertial frame would be the one that has all the axes as straight lines. Only the oblique and rectangular Cartesian coordinate systems satisfy this requirement. Other coordinate systems, for example, a spherical polar coordinate system has as coordinates the polar θ and the azimuthal angle ϕ, which cannot change without violating the rectilinear property in inertial motion.

We see that the first law requires the motion of a free particle, which depends on the definition of force given by the second law.

ii) Law of Causality (2^{nd} Law):

If the total force exerted on a particle by other objects at any specified time is represented by vector F, then-

$$F = ma = dp/dt \qquad \ldots 1$$

where, $a = dv/dt$ is the acceleration of the particle at given instant, m is the mass of the particle, v is the velocity of the particle at that instant and $p=mv$ is the linear momentum.

The vector quantity F is called force and the above equation (1), is taken to be its definition. This law is a complete law.

Newton's original version of the second law also reads somewhat differently: "The change of motion Δv is proportional to the motive force ΔI impressed, and is made in the direction of the right line in which that force is impressed." Thus if we consider all these changes to take place in time Δt, and take the constant to proportionality to be $1/m$, in the limit of $\Delta t \rightarrow 0$, we get the usual form.

Through this law, the study of the motion of bodies became part of a new branch of science called dynamics. For the first time it defined the motion of force through a directly measurable quantity called acceleration and another quantity is called mass. Measurement of mass or the quantity of matter, in a given body would have been extremely difficult. Newton derived the first universal law of force, that is, the inverse square law of the gravitational force, from a combination of Kepler's law of planetary motion and his own three laws of motion, he could then have many situations where he could know the value of mass, acceleration and force, independent of his second law of motion, and therefore test its validity.

The second law is a prescription for formulation the dynamical equations of motion in inertial frames. The first law has already defined what inertial frames are? They are rectangular Cartesian frames in which a free particle either stays at rest or continues with uniform rectilinear motion *ad* infinitum.

However, in real life situations we very often come across various kinds of contact or surface forces, forces produced due to collision, or hindrance to natural motion. Since, Newton's second law demands the a priori knowledge of the total force that a body experience, this force must also include all the forces of reactions that it experiences; in the third law, Newton prescribes the general nature of the force of reaction in relation to the force of action.

iii) Law of Reciprocity (3rd Law):

To the force exerted by every object on a particle, there corresponds an equal and opposite force exerted by the particle on that object.

For two interaction particle, if F_{21} is the force exerted by the first particle on the second, and F_{21} is the force exerted by the second particle on the first, me must have-

$$F_{12} = -F_{21}$$

Using the second law, we have, then

$$\frac{d}{dt}(p_1 + p_2) = 0$$

where, p_1 and p_2 are the linear momenta of the to particles 1 and 2 respectively.

This means that the total linear momentum $p_1 + p_2$ is a constant of motion.

In other words, the total linear momentum of any isolated pair of mutually interaction particles, expressed as a vector sum of quantities p_1 and p_2 is conserved.

iv) Law of Superposition:

The total force F due to several objects acting simultaneously on a particle is equal to the vector sum of the forces F_k, due to each object acting independently, that is,

$$F = \sum_k F_k \qquad \ldots 2$$

This is a "divide and conquer" rule for solving mechanical problems involving complex forces. There is no unique way of dividing the total F into a number of components. In other words, for a given F there is, in general, infinity of solution of equation 2, though, of course, all F_k `s cannot be mutually orthogonal. Newton did not write this law as a separate one, but it is independent of the first three laws, and was first explicitly mentioned by Daniel Bernoulli in 1738.

Newton's laws of motion have been critically examined again over and again, particularly to see whether all the three laws are independent or not. The most widely argued point is that the first law is a special case of the second law, because as we put $F=0$, in eq. 1, it implies that the acceleration a is zero and therefore guarantees rectilinear motion with constant velocity.

Examples of Force Laws and the Corresponding Motions:

From the mechanical standpoint, forces can be divided into two classes: a) Body Force: which act on each point on the body such gravitation, and b) Surface or Contact Forces: acting at the surface only, for example, pressure, tension, elastic forces at the contact, all reactions due to mutual contact between two bodies.

We shall give here a few specific examples of forces

i) Hooke's Law (1675):

Within a certain specified domain of space, the force F acting on a particle is attractive in nature and is linearly proportional to the displacement (r) from the position of equilibrium. That is,

$$F = -kr$$

where, k is the constant of proportionality, called Hooke's constant, and r is measured form the position of equilibrium.

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