VOLUME-05 Part B and C

CONTENTS

I.11. A Group Theory

Introduction to sets, Mapping and binary operations:

I . Sets

Definition of Set:

A set is a collection of objects (called elements or members) of any sort, having some properties in common, e.g. the set of all natural numbers.

A set is finite or infinite according as the number of its elements is finite or infinite. The element of the set is repeated and distinguishable. Here distinct means that no elements of the set are repeated and distinguishable means that given any object whatsoever, it is either in the set or not in the set.

The sets are denoted by braces like { }, e.g. {1,2} and {1,1,2} which represent the same set.

(i) Defining property of a set:

Using any of the notation like, :, | , ∍, s.t. for such that, the defining property of a set is $\{x : P(x)\}$

e.g. a set if even numbers from 2 to 20 may be expressed as,

$$\{x : x = 2n, n = 1,2,....10\}$$

Singleton set: A set having a single element is called as singleton set.

e.g. $\{a\} = \{x : x = a\}$. As another example $\{0\}$ is a singleton set having 0 as the single element.

Null set or void set or empty set: A set having no element is called an empty set and denoted by ϕ, such as

$$\phi = \{x : x \neq x\} = \{\ \}.$$

Subset: Using the notation $\in$ for 'belong to' and $\Rightarrow$ for 'implies', if there are two sets A and B such that every element of A belongs to B,

i.e. $a \in A \Rightarrow a \in B$ (Here, A is called a subset of B or said to be contained in B)

and denoted by,

$A \subset B$ or $B \supset A$ (i.e. B contains A). (Here A is subset of B and B is superset of A)

e.g. {1, 3} is subset of {1, 2, 3} but {1, 2, 3} is superset of {1, 3}.

Equal sets: Two sets A and B are said to be equal if

$$A \subset B \;\; or \; B \subset A$$

e.g. A $=\{a,b,c,d\}$ and $B=\{b,c,a,d\}$ are equal.

The Negations $a \in A, A \subset B$ and $A = B$ are $a \notin A,\; a \not\subset B,$ and $A \neq B,$ respectively.

Axiom of extension: Two sets A and B are equal if and only if they have the same number of elements.

Axiom of specification: If A is a set and $S(x)$ is a condition or statement, then there corresponds a set B whose elements are exactly those elements x of A for which $S(x)$ is true i.e.

$$B=\{x \in A : S(x)\}$$

Axiom of pairing: If a and b are two objects (sets), then there exists a set A such that $a \in A, b \in A$ i.e. $\{x \in A : x = a \; or \; x = b\}$.

Normal and abnormal sets: If a set contains itself as one of its elements, it is said to be an abnormal set, otherwise it is a normal set.

***Equivalent set*:** Two sets A and B are said to be equivalent and denoted by $A \sim B$ if the number of elements of A is equal to the number of elements of B.

Evidently two equal sets are equivalent but the converse is not true.

***Proper subset*:** A set A is said to be the proper subset of B and denoted by $A \subset B$, if every element of A is an element of B and there is atleast one element of B which is not the element of A so that $A \neq B$.

e.g. $\{1,3,5\}$ is a subset of $\{1,3,5\}$ but it is not its proper subset while $\{1,3,5\}$ is a proper subset of $\{1,3,5,7\}$.(some authors use $\subseteq$ for a subset and $\subset$ for a proper subset.)

Axiom of power set: The power set of a set A is the family or class of all the subsets of A and denoted by, $P(A)$.

e.g. if $A=\{1,2,3\}$

then $P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Obviously when A consists of 3 elements, $P(A)$ consists of 2^3 elements.

In general if A consists of *m* elements, $P(A)$ will consist of 2^m elements.

The power set of *A* is also denoted by 2^A.

(ii) Operation on Sets:

***Union*:** The union of two sets *A* and *B* denoted by $A \cup B$ and read as '*A* union *B*' is the set of all objects which are members of *A* or *B* (or both).

i.e. $A \cup B = \{x \mid x \in A \; or \; x \in B\}$

The word 'or' used here gives the inclusive sense and/or.

e.g. if $A = \{1,2,3\}$, $B = \{4,5,6\}$, then $A \cup B = \{1,2,34,5,6\}$.

The union of n sets $A_1, A_2 ..... A_n$ is defined as

$$A_1 \cup A_2 \cup ...... A_n = \bigcup_{i=1}^{n} A_i = \{x : x \in A_i \text{ for some } i \text{ in the range, } i = 1 \text{ to } i = n\}$$

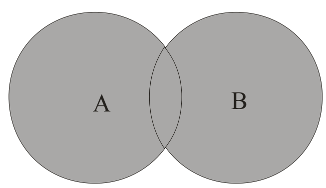

Figure 1

Intersection: The intersection of two sets A and B denoted by $A \cap B$ and read as '*A* intersection *B*' is the set of all objects which are members of both *A* and of *B* i.e.

$A \cap B = \{x \mid x \in A \; or \; x \in B\}$

e.g. if $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, then $A \cap B = \{2,4\}$.

The intersection of *n* sets $A_1, A_2 ..... A_n$ is defined as

$$A_1 \cap A_2 \cap ..... \cap A_n = \bigcap_{i=1}^{n} A_i = \{x : x \in A_i \text{ for } 1 \le i \le n\}$$

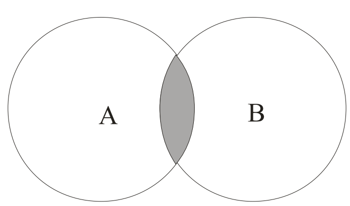

Figure 2

***Family or class or collection of sets*:** If a set consists of elements which are sets themselves, then such a set is called as 'family or class' of sets.

e.g. a set $A^* = \{\{1\}, \{1,2\}, \{1,2,3\}\}$.

***Indexed family of sets*:** For a given index set Λ, a collection of sets such that to each member of A there corresponds a member of the collection of the sets, is known as indexed family of sets and written, as

$$\mathbf{A^*} = \{A_\alpha : \alpha \in \Lambda\}$$

where, $\alpha \in \Lambda$ is an index and

A_α denotes the indexed sets.

The arbitary union of sets $\{A_\alpha : \alpha \in \Lambda\}$ is given by

$$\cup\{A_\alpha : \alpha \in \Lambda\} = \{x : x \in A_\alpha \text{ for at least one } \alpha \in \Lambda\}$$

If $\Lambda = \phi, \cup\{A_\alpha : \alpha \in \phi\} = \phi$.

The arbitrary Intersection of sets $\{A_\alpha : \alpha \in \Lambda\}$ is given by

$$\cup\{A_\alpha : \alpha \in \Lambda\} = \{x : x \in A_\alpha \forall\, \alpha \in \Lambda\},$$

where, $\forall$ is the notation for 'for every'.

If $\Lambda = \phi$, then $\cup\{A_\alpha : \alpha \in \phi\} = U$.

***Mutually exclusive or disjoint sets*:** If there are two sets A and B such that $A \cap B = \phi$, then A and B are said to be disjoint.

e.g., If $A = \{1, 2\}, B = \{5, 6\}$ then $A \cap B = \phi$.

Universal set: All the sets under consideration are assumed to be the subsets of some fixed set called as the universal set and denoted by U.

Continued with...Page 5 Onwards.... It's So Goooood!!!, Buy it now...!