VOLUME-01 Part B and C

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I. **Mathematical Methods of Physics**

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I.1. Dimensional Analysis: Vector Algebra and Vector Calculus

1. Kinds of Vectors

Equal Vectors. Two given vectors may be equal only when they have the same magnitude and the same direction i.e., the given two vectors are equal if and only if they have the same or parallel support with equal length and the same sense.

For example

$$\mathbf{V}\left(=\overrightarrow{OP}\right)=\mathbf{V}_1\left(=\overrightarrow{O'P'}\right)=-\mathbf{V}_2\left(=\overrightarrow{O''P''}\right)$$

where $\mathbf{V}_1$ and $\mathbf{V}_2$ have the same scalar magnitude as $\mathbf{V}$ and $\mathbf{V}_1$ has the same and $\mathbf{V}_2$ the opposite sense to that of $\mathbf{V}$.

Null Vector: A vector having the initial and the terminal points coincident is termed as a zero vector or a null vector. Thus a null vector has its module zero.

Unit Vector: A vector having its modulus as unity is called a unit vector.

If $\mathbf{a}$ is a vector and 'a' its modulus, then unit vector $\mathbf{a}$ is denoted by $\hat{\mathbf{a}}$ (read as 'a hat' or 'a caret') defined as,

$$\hat{\mathbf{a}}=\frac{\mathbf{a}}{|\mathbf{a}|}=\frac{\mathbf{a}}{a}$$

Polar vectors: The line vectors representing the quantities like force, velocity etc., in which merely a linear action in a particular direction is involved, are termed as polar vectors.

Axial Vectors: The line vectors representing the quantities like angular velocity angular acceleration etc., in which some rotational action is involved about an axis and which are drawn parallel to the axis of rotation in order that the magnitude of the quantity is determined by the length of the vector and the direction by the rule of right handed screw (i.e., rotation being considered in clockwise direction), are termed as axial vectors.

Free Vector: Evidently a vector can be represented by an infinite number of equal vectors by drawing parallel supports. Such a vector which can be transported from place to place such that it remains of the same magnitude and keeps up the same direction is termed as a free vector. In fact a free vector is assumed to remain the same through transportation, irrespective of its position in space.

Localised or line vector: We have defined that the value of a free vector depends only on its length and direction, but it depends also on its position in space, i.e., if a vector is restricted to pass through a given origin, then it is termed as a localized vector.

Collinear Vectors: The vectors parallel to the same line, regardless of their magnitudes and sense of directions are termed as collinear vectors. In other words the vectors having the same or parallel support are known as collinear vectors, such vectors are parallel to each other and they may coincide in a special case. As such there exists a scalar ratio say λ between any two collinear vectors **a** and **b** of the form.

$$\mathbf{b} = \lambda \mathbf{a}$$

which follows that one of the two collinear can be expressed as the scalar multiple of the other.

Non-collinear vectors: The vectors whose directions are neither parallel nor coincident are said to be non-collinear.

Like Vectors or co-directional vectors: The vectors which are collinear and have the same sense of directions i.e. the vectors directed in the same sense irrespective of their magnitudes are termed as like vectors.

Unlike Vectors: The vectors which are collinear but have opposite sense of directions from each other are termed as unlike vectors.

Coplanar Vectors: A system of vector lying in the parallel planes or which can be made to lie in the same plane are said to be coplanar vectors. Evidently any two vectors are always coplanar.

Non-coplanar Vectors: A system of vectors consisting of three or more vectors which cannot be made to lie in the same plane are called non-coplanar vectors.

Reciprocal Vector: Any vector having its direction the same as that of a given vector **a**, but its magnitude as the reciprocal of the magnitude of **a** is termed as the reciprocal vector of **a** and written as $\mathbf{a}^{-1}$ or $\frac{1}{\mathbf{a}}$.

As such

$$\mathbf{a}^{-1} = \frac{1}{a}\hat{\mathbf{a}}$$

$$\mathbf{a}^{-1} = \frac{a}{a^2}\hat{\mathbf{a}} \qquad \text{(by definition of a unit vector).}$$

$$\mathbf{a}^{-1} = \frac{\mathbf{a}}{a^2}$$

In this connection it is notable that the magnitude and so the reciprocal of the magnitude of a unit vector being unity, the unit vector is reciprocal to itself and it is said to be self-reciprocal.

Negative Vector: The vector having the same magnitude as the vector **a** but opposite direction, is known as the negative of **a** and written as **–a**.

Position Vector: If a vector $\overrightarrow{OP}$ specifies the position of a point relative to an arbitrarily chosen point *O*, then $\overrightarrow{OP}$ is called the Position vector of *P* with respect to *O*, the origin of vectors.

Problem 1. If $\{\mathbf{a},\mathbf{b},\mathbf{c}\}$ is a right handed set, which of the following sets are right handed?

(i) **a**, **c**, **b**; (ii) **b**, **c**, **a**; (iii) **b**, **a**, **c**; (iv) **c**, **a**, **b**; (v) **c**, **b**, **a**

It is clear that the sets (ii) and (iv), i.e., **b**, **c**, **a**; and **c**, **a**, **b** are right handed.

Problem 2. Discuss the geometrical significance of $a\mathbf{A}+b\mathbf{B}=\mathbf{0}.$

We have $a\mathbf{A}+b\mathbf{B}=\mathbf{0}.$ a, b being scalars.

This can be written as,

$$\mathbf{A}=-\frac{b}{a}\mathbf{B}$$

$$=\lambda\mathbf{B} \qquad \text{if } \lambda=-\frac{b}{a}$$

i.e. **A** is expressible as a scalar multiple of **B** so that the vectors **A** and **B** are parallel or collinear.

2. Addition of Vectors:

The characterization of process of summation is inherited in the composition of two or more displacement of a point. Suppose that we have two vectors **a** and **b** acting at a point *O* as shown in Fig.1.

Let $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OB}=\mathbf{b}.$

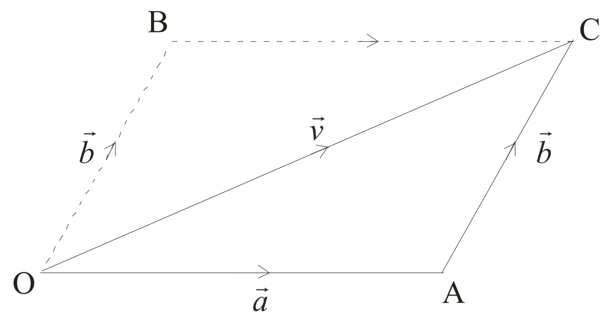

Figure 1

Clearly the resultant effect of the vectors **a** and **b** is the same as that of their vector sum **v** obtained by setting off the vector **b** at the end of a and then joining the beginning of **a** to the end of **b**. This geometrical construction utilized to find the vector sum of two vectors **a** and **b** is known as the parallelogram law of addition of vectors.

$$\begin{aligned}\mathbf{v} &= \overrightarrow{OC} \\ &= \overrightarrow{OA} + \overrightarrow{AC} \\ &= \mathbf{a} + \mathbf{b}\end{aligned} \qquad \ldots 1$$

A similar result follows by starting with **b** and setting off the vector **a** on **b**, i.e.,

$$\begin{aligned}\mathbf{v} &= \overrightarrow{OC} \\ &= \overrightarrow{OB} + \overrightarrow{BC} \\ &= \mathbf{b} + \mathbf{a}\end{aligned} \qquad \ldots 2$$

Conclusively the result of adding two co-initial vectors is the vector represented by the diagonal of the parallelogram having the two given vectors as its adjacent sides.

From (1) and (2) it follows that,

$$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$$

i.e., the two vectors obey the commutative law of addition, according to which the vector sum of two vectors is independent of their order.